ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

FREE VIBRATION OF RHOMBIC PLATE WITH VARYING THICKNESS IN 2D AND THERMAL EFFECT IN X-DIRECTION

Subodh Kumar Sharma
Dept. of Mathematics Govt. P.G College, Ambala Cantt., Haryana, INDIA

ABSTRACT

Rapid advancements made in materials science and manufacturing technologies has lead to the application of composite materials in various areas of engineering applications like aerospace industries, marine structures, automobiles, power plant systems etc. The object of this paper is to investigate the vibration problem of rhombic plate with 2D linearly varying thickness and Thermal effect linearly in x-direction. Rayleigh-Ritz technique is used to obtain a frequency equation with a two-term deflection function. Numeric values of frequencies are calculated for different values of thermal gradient, taper parameter and skew angle for both the modes of vibration. The results are presented in tabular forms.

Keywords: variable thickness, skew-angle, rhombic plate, taper parameter, thermal gradient.

INTRODUCTION

In modern technology an interest towards the effect of high temperatures on rhombic plates of variable thickness is developed due to applications in various engineering branches such as nuclear, power plants, aeronautical, chemical etc. where metals and their alloys exhibits visco-elastic behavior. Therefore for these changes the structures are exposed to high intensity, heat fluxes and material properties undergo significant changes. The materials are being developed, depending upon the necessity and stability, so that these can be used to give better strength, elasticity, weight effectiveness and good organization. So some new materials and alloys are utilized in making structural parts of equipment used in modern technological industries like jet engine, telephone industry, space craft, etc. Applications of such materials are due to reduction of weight and size, low expenses and enhancement in effectiveness and strength. It is well known that first few frequencies of structure should be known before finalizing the design of a structure. The study of vibration of plate structures is important in a wide variety of applications in engineering design. Elastic plates are widely employed nowadays in civil, aeronautical and marine structures designs. Complex shapes with variety of thickness variation are sometimes incorporated to reduce costly material, lighten the loads, and provide ventilation and to alter the resonant frequencies of the structures. Dynamic behavior of these structures is strongly dependent on boundary conditions, geometric shapes, material properties etc. As technology develops new discoveries day by day like in jet engine, field of spacecraft and nuclear power plants etc., the time dependent behavior of materials has become of great importance. Thus, the need of the study of vibration of visco-elastic plates (it may be rectangular, circular, elliptical etc.) of certain aspect ratios with some simple boundary conditions has been increased rapidly. The frequency to the first two modes of vibration is obtained for a clamped rhombic plate for various values of thermal gradient (α), taper constant (β) and skew angle (θ). All the results are shown in table and graphical form.

METHODLOGY

The rhombic and oblique co-ordinates are related as follow:

$$\left.\begin{aligned} x' &= x - y\tan\theta \\ \text{and}\quad y' &= y\sec\theta \end{aligned}\right\} \qquad (1)$$

the boundaries of the plate in oblique co-ordinate are

$$x' = 0, x' = a \text{ and } y' = 0, y' = b \qquad (2)$$

It is assumed that the rhombic plate is subjected to a study one dimensional temperature distribution along the length, i.e. in x-direction,

$$\tau = \tau_0\left(1 - \frac{x'}{a}\right) \qquad (3)$$

where τ denotes the temperature excess above the reference temperature at any point at a distance $\frac{x\prime}{a}$ and τ_0

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

denotes the temperature excess above the reference temperature at the end $x' = a$ .The temperature dependence of the modulus of elasticity is given by

$$E(\tau) = E_0(1-\gamma\tau) \qquad (4)$$

where E_0 is the Young modulus at the reference temperature i.e. at $\tau = 0$ with the temperature at the end of the plate as reference. The Young's modulus in view of equation (3) and (4) become

$$E(y') = E_0(1-\alpha(1-\frac{x'}{a})) \qquad (5)$$

where $\alpha = \gamma\tau_0 (0 \le \alpha < 1)$

Variation in Thickness

Thickness variation of the plate is assumed to be linear in x and y directions i.e.

$$h = h_0(1+\beta_1\frac{x'}{a})(1+\beta_2\frac{y'}{b}) \qquad (6)$$

Method of Solution

Using the Rayleigh-Ritz technique, one requires that the maximum strain energy must be equal to the maximum kinetic energy. It is, therefore, necessary for the problem under consideration that

$$\delta(V-\lambda^2 T) = 0 \qquad (7)$$

The maximum kinetic energy, V in the plate when it is executing transverse vibration mode shape $W(x', y')$ are [1]:

$$T = \frac{1}{2}\rho\omega^2\cos\theta\iint hW^2 dx'dy' \qquad (8)$$

And

$$V = \frac{1}{2\cos^3\theta}\iint D[W^2{}_{,x'x'} - 4\sin\theta W_{,x'x'}W_{,x'y'} + 2(\sin^2\theta + \nu\cos^2\theta)W_{,x'x'}W_{,x'y'} + 2(1+\sin^2\theta - \nu\cos^2\theta)W_{,x'y'}{}^2 - 4\sin\theta W_{,x'y'}W_{,y'y'}{}^2]dx'dy' \qquad (9)$$

For arbitrary variations of W satisfying relevant geometric boundary conditions. For a rhombic plate clamped along all four edges the boundary conditions are

$$W = W_{,x'} = 0 \; at \; x' = 0, a$$

and $$W = W_{,y'} = 0 \; at \; y' = 0, b \qquad (10)$$

And corresponding two term deflection function is taken as

$$W(x', y') = (\frac{x'^2}{a^2})(\frac{y'^2}{b^2})(1-\frac{y'}{b})^2[A_1 + A_2(\frac{x'}{a})(\frac{y'}{b})(1-\frac{x'}{a})(1-\frac{y'}{b})] \qquad (11)$$

Using eq^n (8) & (9) in eq^n (7) one gets

$$\delta(V^* - \lambda^2 T^*) = 0, \ldots\ldots\ldots n = 1, 2 \qquad (12)$$

where

$$V^* = \frac{1}{\cos^4\theta}\int_0^a\int_0^b (1-\alpha(1-\frac{x'}{a}))[(1+\beta_1\frac{x'}{a})(1+\beta_2\frac{y'}{b})]^3 [W_{,x'x'^2} - 4\sin\theta W_{,x'x'}W_{,x'y'} + 2(\sin^2\theta + \nu\cos^2\theta)W_{,x'x'}W_{,y'y'} + 2(1+\sin^2\theta - \nu\cos^2\theta)W_{,x'y'}{}^2 - 4\sin\theta W_{,x'y'}W_{,y'y'} + W_{,y'y'}{}^2]dx'dy'.$$

and

$$T^* = \frac{1}{2}\rho\omega^2\int_0^a\int_0^b hW^2 dx'dy'$$

and $$\lambda^2 = \frac{12a^4\omega^2\rho(1-\nu^2)}{E_0 h_0^2}$$

Eq^n (12) involves unknown constant A_1 & A_2 arising due to substitution of $W(x', y')$ from eqn (11). These unknowns are to be determined from (1) for which

$$\frac{\partial}{\partial A_n}(V^* - \lambda^2 T^*) = 0, \quad n = 1, 2 \qquad (13)$$

On simplifying one gets

$$c_{n1}A_1 + c_{n2}A_2 = 0 \qquad n = 1, 2 \qquad (14)$$

where c_{11} , $c_{12}(=c_{21})$, c_{22} involve parametric constants and frequency parameter.

For a non-trivial solution the determinant of the coefficients of equation (14) must be zero i.e.

$$\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} \qquad (15)$$

Frequency equation (15) is quadratic in λ^2, so it will give two roots. These two values represent the two modes of vibration of frequency i.e. λ_1 & λ_2 for different values of skew angle and thermal gradient. From equation (15) one can easily obtain frequency for both the mode.

RESULT AND DISCUSSION

All computation has been done for frequencies of visco-elastic rhombic plate for different values of thermal gradient (α), taper constant (β) and skew angle (θ) for the first two modes of vibration.

Table 1:-It is clearly seen that frequency decreases as the thermal gradient increases from 0.0 to 0.8 for $\beta_1 = \beta_2 = 0.0$ and $\beta_1 = \beta_2 = 0.4$ & θ=30° for both modes of vibration.

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

Table 2 :- It is evident that value of frequency increases as the value of skew angle increases from 0° to 60° for $\beta_1 = \beta_2 = 0.0$ and $\beta_1 = \beta_2 = 0.4$ at fix **α** =0.0.

Table 1:-Frequency vs thermal gradient with

α	$\beta_1 = \beta_2 = 0.0$ ModeI	Mode II	$\beta_1 = \beta_2 = 0.4$ Mode I	Mode II
0	195.12	50.01	232.66	57.77
0.2	182.72	43.70	224.72	56.59
0.4	173.51	41.44	215.68	54.83
0.6	163.94	39.37	207.91	52.52
0.8	152.99	36.98	198.69	50.68

θ=30

Graph 1:-Frequency vs thermal gradient with θ=30

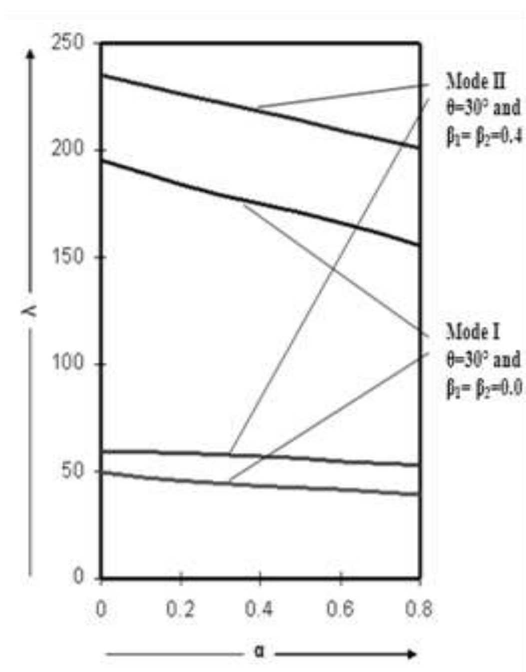
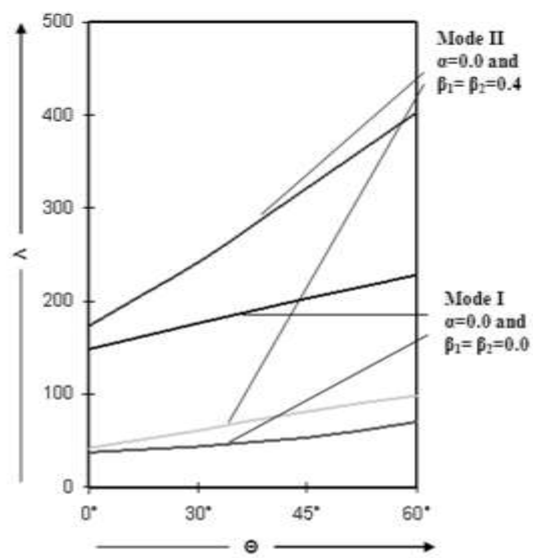


Table 2:-Frequency v/s skew angle with α=0.0

Θ	$\beta_1 = \beta_2 = 0.0$ & α =0.0 Mode I	Mode II	$\beta_1 = \beta_2 = 0.4$ & α =0.0 Mode I	Mode II
0°	149.21	37.31	172.13	40.75
30°	175.01	42.20	240.52	58.82
45°	295.52	71.90	368.10	89.65
60°	599.66	148.35	760.50	188.38

Graph 2:-Frequency v/s skew angle with α=0.0



CONCLUSION

Our aim is to provide such kind of a mathematical design so that scientist can perceive their potential in mechanical engineering field & increase strength, durability and efficiency of mechanical design and structuring with a practical approach .Actually this is the need of the hour to develop more but authentic mathematical model for the help of mechanical engineers/researchers/practitioners .Therefore mechanical engineers and technocrats are advised to study and get the practical importance of the present paper and to provide much better structure and machines with more safety and economy.

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

REFERENCES

[1]. A.W.Leissa, "Vibration of plate", NASA SP-160, 1969.

[2]. A.W.Leissa, "Recent studies in plate vibration 1981-1985 part II", complicating effects, Shock and Vibration Dig., 19(1987), 10-24.

[3]. Khanna, A., Kaur, N., & Sharma, A. K. "Effect of varying poisson ratio on thermally induced vibrations of non-homogeneous rectangular plate". Indian Journal of Science and Technology, 5(9), 2012, 3263-3267.

[4]. Sharma, S. K., & Sharma, A. K, "Mechanical Vibration of Orthotropic Rectangular Plate with 2D Linearly Varying Thickness and Thermal Effect", International Journal of Research in Advent Technology, 2(6), 2014, 184-190.

[5]. Khanna, A., & Sharma, A. K, "Vibration Analysis of Visco-Elastic Square Plate of Variable Thickness with Thermal Gradient", International Journal of Engineering and Applied Sciences, Turkey, 3(4), 2011, 1-6.

[6]. Kumar Sharma, A., & Sharma, S. K, " Vibration Computational of Visco-Elastic Plate with Sinusoidal Thickness Variation and Linearly Thermal effect in 2D", Journal of Advanced Research in Applied Mechanics & Computational Fluid Dynamics, 1(1), 2014, 46-54.

[7]. Khanna, A., Kumar, A., & Bhatia, M, "A Computational Prediction on Two Dimensional Thermal Effect on Vibration of Visco-elastic Square Plate of Variable Thickness", Presented and Published in Proceeding of CONIAPS XIII, 2011, held in UPES, Deharadun.

[8]. Khanna, A., & Sharma, A. K, "Natural Vibration of Visco-Elastic Plate of Varying Thickness with Thermal Effect", Journal of Applied Science and Engineering, 16(2), 2013, 135-140.

[9]. Kumar Sharma, A., & Sharma, S. K, "Free Vibration Analysis of Visco-elastic Orthotropic Rectangular plate with Bi-Parabolic Thermal Effect and Bi-Linear Thickness Variation," Journal of Advanced Research in Applied Mechanics & Computational Fluid Dynamics, 1(1), 2014, 10-23.

[10]. Sharma, S. K., & Sharma, A. K, "Effect of Bi-Parabolic Thermal and Thickness Variation on Vibration of Visco-Elastic Orthotropic Rectangular Plate", Journal of Advanced Research in Manufacturing, Material Science & Metallurgical Engineering, 1(2), 2014, 26-38.

[11]. Khanna, A., & Sharma, A. K, "Mechanical Vibration of Visco-Elastic Plate with Thickness Variation", International Journal of Applied Mathematical Research, 1(2), 2012, 150-158.

[12]. Khanna, A., Kaur, N., & Sharma, A. K. "Effect of varying poisson ratio on thermally induced vibrations of non-homogeneous rectangular plate," Indian Journal of Science and Technology, 5(9), 2012, 3263-3267.

[13]. Khanna, A., & Sharma, A. K, "Analysis of free vibrations of VISCO elastic square plate of variable thickness with temperature effect", International Journal of Applied Engineering Research, 2(2), 2011, 312-317.

[14]. Sharma, S. K., & Sharma, A. K, "Vibration Computational of Visco-Elastic Plate with Sinusoidal Thickness Variation and Linearly Thermal effect in 2D", Journal of Advanced Research in Applied Mechanics & Computational Fluid Dynamics, 1(1), 2014,9-18.

Photo	Biography
	**Dr. Subodh Kumar Sharma** received PhD degree in Mathematics from C.C.S University Meerut, INDIA. Presently working in Govt. P.G College, Ambala Cantt., Haryana, INDIA and has 32 years of teaching experience. His current research interests includes elastic vibration, solid mechanics, plate vibration, fluid mechanics etc. In his research he has published 20 research papers in reputed International Journals and 3 books at International Level. He also visited to many countries for guest lectures.